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AN EXPERIMENTAL ELECTRON ACCELERATOR

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Abstract

A description is given of an electron accelerator which depends on a magnetic resonance principle. It was built to test the practicability of using iron-free magnetic fields and to indicate possible advantages of a magnetic resonance method for producing high energy electrons. The apparatus was successful in producing X-rays of a mean energy of 60 kV.

Introduction

The problem of the acceleration of atomic ions and electrons has been a vital one since nuclear disintegrations were first artificially achieved by Cockcroft and Walton in 1932. Many successful methods have been subsequently developed, but interest has now turned to extending these methods to high energies, of the order of 1000 million electron volts.

To consider the acceleration of electrons only, the method of magnetic induction acceleration, commonly referred to as the ' betatron ' method, was until recently the only method which had been used to produce electrons of energy greater than several million volts, and although to date a machine has been made to produce electrons of 100 million volts it is generally considered that the enormous size of the magnet required for the generation of 1000 MeV electrons would prohibit its construction.

The application of the method of magnetic resonance acceleration, referred to as the ' cyclotron ' method, to electron acceleration has until recently not been considered feasible on account of the relativity mass increase of electrons at high energies. In 1945-46 suggestions were made by Veksler and McMillan that a magnetic resonance method might be used if either the magnetic field or accelerating electric field frequency were slowly varied during acceleration. This method, which is now widely referred to as the ' synchrotron ' method, also possesses an advantage, as compared with the betatron method, that only a peripheral field may be required. The construction of a synchrotron to produce 1000 MeV electrons with a magnet of practicable proportions appears feasible and many designs of such generators have already been laid down.

In all ' cyclical ' generators the maximum attainable electron energy is determined by the product of the outer orbital magnetic field and the radius of curvature of the orbit, and when iron is used to produce the magnetic field, high energy electrons can only be obtained by increasing the orbital radius since the flux density in iron saturates at approximately 15,000 gauss.

The purpose of the iron is, of course, to reduce the number of ampere-turns required to produce the magnetic field. It seems worthwhile, however, to consider the use of air-cored fields, since the elimination of gigantic size iron yokes would have tremendous advantages in both construction and expense.

The field B_1 in a gap of length d in an iron yoke is given by :

$$B_1 = \frac{\mu_0 \cdot 4\pi \mathrm{Ni}}{\left(d + \frac{\mu_0}{\mu} \cdot l\right)}$$

where N is the number of turns in the energizing coil carrying current i.

The field B_A in the central plane of a Helmholtz system of air-cored coils of radius a is given by :

$$B_A = \frac{\mu_0 \cdot 32\pi Ni}{5\sqrt{5} \cdot 2a}$$

where N is again the total number of turns in the two energizing coils.

For the same field, therefore, the ratio of the number of ampere-turns required in the case of air-cored coils to the case of iron-cored coils is $\frac{8a}{2 \cdot 85\ d}$.

This ratio clearly depends upon the ratio of the extent of the field to its length and we can gain some estimate of it from an actual cyclotron. For example, a cyclotron generating 2 million volt protons had a field gap in iron of 16 inches radius and 3 inches length. Thus, in order to produce this same field by means of air-cored coils an energizing system of approximately $7\frac{1}{2}$ times the ampere-turns would be required.

If the above figure is taken as a guide to the relative magneto-motive forces required for producing cyclotron fields by air and iron cores, and if it is remembered that in synchrotron operation the power dissipated in the coils is less than for continuous operation, it is seen that the possibilities of using air-cored coils are favourable.

The present work was undertaken to test both the possibility of using air-cored fields and the applicability of the method of magnetic resonance acceleration to electron acceleration. The feasibility of constructing an experimental model was first presented when sources of high power ultra-high-frequency electromagnetic radiations became available. During the years 1942-1945 work was being done in the Melbourne Physics Laboratory on magnetron oscillators and these techniques were carried over to the problems of electron acceleration.

Theory

The motion of an electron of charge e and rest mass m_0 in a magnetic field B is described in Gaussian units by the equation :

$$\frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} \cdot \frac{v^2}{r} = \frac{Bev}{c} \quad \ldots \quad (1)$$

where r is the radius of curvature of the electron's orbit for a velocity v (c=velocity of light).

The frequency of revolution υ of the electron in its orbit is given by:

$$\upsilon = \frac{Be}{2\pi m_0 c}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \quad \ldots \quad (2)$$

If the electron is to be accelerated by a constant frequency electric field, the following condition, obtained from (1) and (2), must therefore be satisfied.

$$B = \frac{B_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} \quad \ldots \quad (3)$$

The resonance magnetic field B_0, when v is small compared with c, is given by :

$$B_0 = \frac{2\pi c \upsilon m_0}{e} \quad \ldots \quad (4)$$

The electron energy E is related to the velocity v by the relativity relation :

$$E = \frac{m_0c^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} - m_0c^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

If it is considered that E increases at the uniform rate of 2Ve per revolution, where V is the dee voltage, then by (3) and (4), B should increase at the rate given by the equation :

$$B = B_0 \left\{ \frac{2Vev}{m_0c^2} \cdot t + 1 \right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

in order that the electron should remain in phase with the accelerating field.

For a field increasing at this rate, however, there is also a magnetic induction effect, and the energy F added per turn due to this effect is :

$$F = \frac{e}{c} \cdot \frac{d}{dt} (\pi r^2 B) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

This energy may be evaluated approximately by assuming that the electron has reached its maximum orbital radius given by :

$$r_\infty = \frac{c}{2\pi v} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

In actual fact the orbital radius soon tends to this value on account of the rapid approach of the electron velocity towards the velocity of light. Thus, assuming that B increases according to equation (6), the energy added per turn from the betatron effect is calculated to be Ve. This energy is to be added (the direction of the electromotive force is correct) to the energy gained from the cyclotron effect, and therefore the field B should increase at a faster rate than that given by equation (6) if the electron is to be kept in phase with the accelerating field. By the method of successive approximations it is clear that if B increases at the rate given by :

$$B = B_0 \left\{ \frac{4Vev}{m_0c^2} \cdot t + 1 \right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

then the electron and the accelerating field will be in phase, and the energy added by both betatron and cyclotron effects will be 2Ve per turn.

The rate of change of field given by equation (9) will also satisfy the betatron condition for constant radius orbit. In the betatron, in order to maintain a constant radius orbit, the rate of change of average flux density threading the orbit is twice that at the orbit itself. The same condition can be satisfied if the flux density threading the orbit is the same as that at the edge, but the energy of the electron increases at twice the rate determined by the magnetic induction effect. This is just the case outlined by equation (9).

If the rate of change of field is greater than that given by equation (9) it can be shown from elementary considerations of phasing that the electron will not accelerate constantly. Taking into account, therefore, acceleration by both cyclotron and betatron effects, constant acceleration occurs for :

$$\frac{dB}{dt} < \frac{4Vev}{m_0c^2} \cdot B_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

For a rate of increase of field slower than that given by equation (10), the acceleration corresponds more closely to the synchrotron principle, considered by Veksler and MacMillan. The electron will then accelerate constantly but will lag behind the accelerating field. If the field increases so slowly that the betatron effect is small compared with the cyclotron effect, according to Veksler, stable electron acceleration is obtained for :

$$\frac{dB}{dt} \ll \frac{2Vev}{m_0c^2} \cdot B_0 \qquad \text{(11)}$$

In the synchrotron type of acceleration, the electron will exist in a state of equilibrium energy E_0, where :

$$E_0 = \frac{c}{2\pi v} \cdot B \qquad \text{(12)}$$

This energy state corresponds to the condition in which the electron is 90° in phase behind the accelerating field, and stable energy oscillations will occur about this energy. The orbital radius of curvature r of an electron moving with a velocity v corresponding to an equilibrium energy is :

$$r = \frac{\sqrt{E^2_0 - m_0c^2}}{B} \qquad \text{(13)}$$

In Table I are shown the variations of magnetic fields and orbital radii of electron accelerators using different frequencies for the accelerating field. The static fields B_0 are calculated from equation (4) and the maximum orbital radii from equation (8). The values of the final magnetic field required to accelerate electrons to 10 and 100 million volts have been calculated from the following expression derived from (3) and (5) :

$$B = B_0 \left(\frac{E + m_0c^2}{m_0c^2} \right) \qquad \text{(14)}$$

These values of r and B tend, for $m_0c^2 < E$, to those obtainable for synchrotron operation.

TABLE I—VARIATION OF ACCELERATOR CONSTANTS

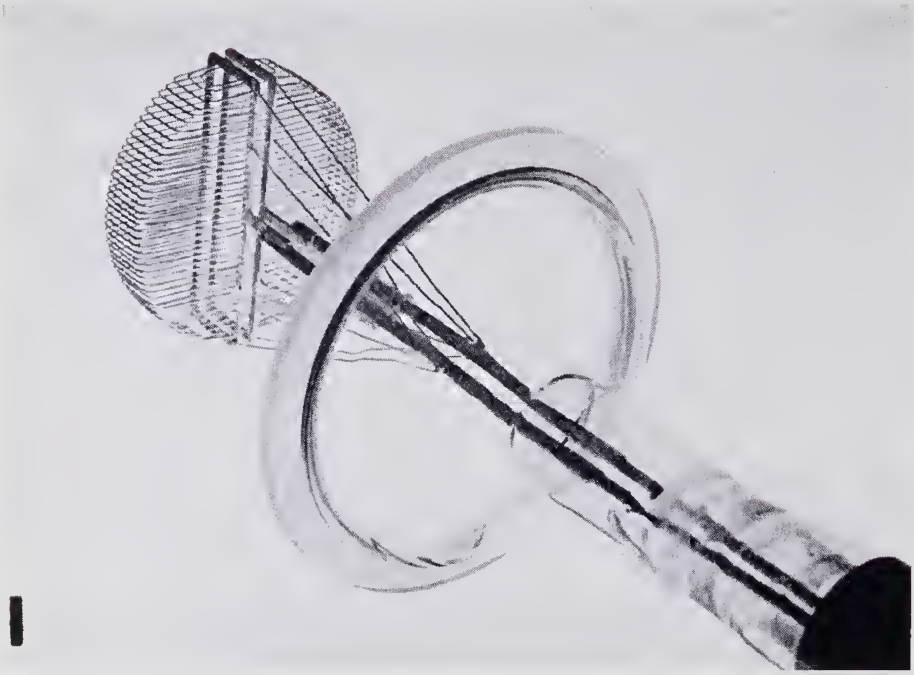
Frequency (megacycles / sec.)	3,000	1,200	300	30	12
Static field, B_0 (gauss) ..	1,060	425	106	10	4
Max. field, B, at 10 MeV. ..	22,500	8,900	2,250	225	90
Max. field, B, at 100 MeV	213,000	85,300	21,300	2,130	850
Max. radius, r_∞ (cm) ..	1·6	4	16	160	400

It is clear from the table that the fields are smaller and the orbital sizes are larger the lower are the accelerating frequencies. For very high energies it is likely that the use of large orbits and fields of relatively small magnitudes would prove most favourable, but for the present experimental test it was convenient to use a frequency of 1,200 megacycles/sec. for which the orbital size was of readily manageable proportions.

General Description of Accelerator

The cyclotron method of accelerating atomic particles requires essentially two electrodes or ' dees '. These are the halves of a split shallow cylindrical box and are situated in a magnetic field, the direction of which is along the axis of the cylinder. The accelerating electric field occurs across the gap between the dees and is created by a source of high frequency electrical oscillations connected to the dees. The electrons describe helical orbits in the central plane of the dees. The problem of generating the high frequency oscillations and of feeding the dees is discussed in the next section on the magnetron oscillator.

The static magnetic field B_0 required to resonate the electrons with the accelerating field when the electron energies are low, was developed from a



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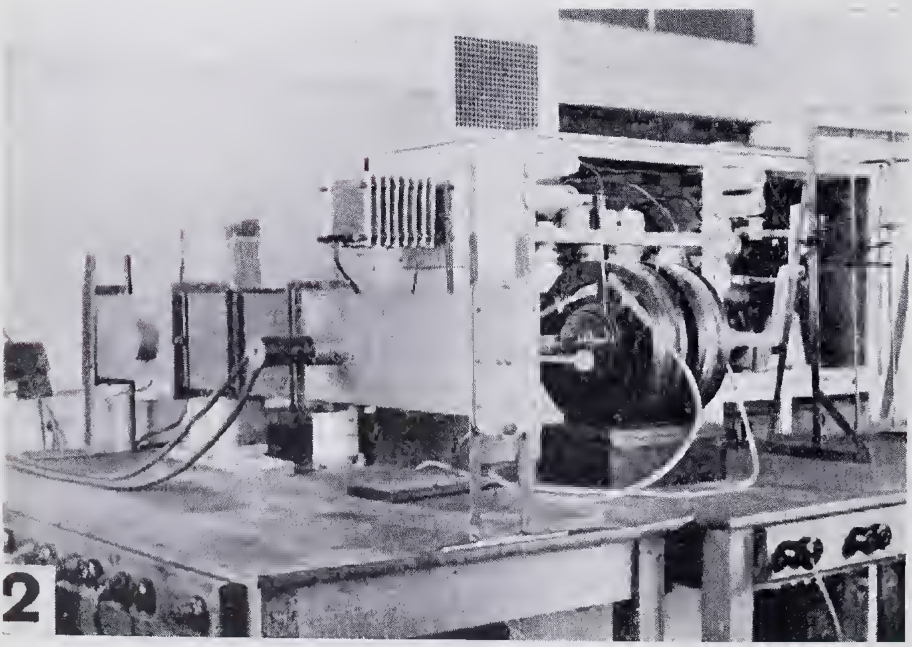

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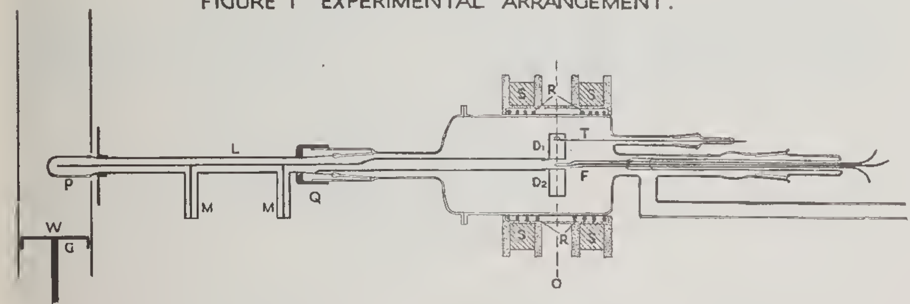
Fig. 1. Construction of the 'Dees'
Fig. 2. General Experimental Arrangement

pair of Helmholtz air-cored coils. Further details of the field are given in a later section.

In an attempt to test the synchrotron principle, a growing magnetic field, referred to as the dynamic field, was superimposed on the static field at the instant of start of the accelerating field pulse. This dynamic field was produced by discharging a condenser through another pair of Helmholtz coils of low inductance and low loss. Further details of this field are also given later.

FIGURE I EXPERIMENTAL ARRANGEMENT.



The accelerator itself is best described by a brief reference to Fig. 1. A probe P of a concentric line L is matched into a waveguide W by moving the plunger G. The line L, which has two shorted quarter-wave stubs M, M, to support the centre conductor, takes the high-frequency power to the dees D_1 and D_2. At Q the concentric line changes to a pair of parallel Lecher wires which fan out to the width of the dees at their ends. This is shown clearly in Plate VIII, Fig. 1. The dees are approximately 5 cm. in diameter and 2 cm. in depth, and are constructed from wire hoops. The aim of this construction is to form a continuous shield of the electron orbits for the high frequency electric field and yet allow the penetration of the increasing dynamic magnetic field. The edges of the dees are separated at the ' gap ' by approximately 6 mm. A filament stem F is situated along the axis of the generator and a single hair-pin tungsten filament lies in the orbital plane O. The filament is partly enclosed in an earthed split-cylinder so that the electrons are emitted only into one dee. A target T is situated in the orbital plane towards the outer edge of one of the dees. The whole dee-structure is enclosed in a glass envelope E which is evacuated by a two-stage oil diffusion pump to a pressure of the order of 10^{-5} mm. Hg. The dees are mounted in the mid-plane of the Helmholtz coil system formed by the static field coils SS and the dynamic field coils RR.

A photograph of the general experimental arrangement is shown in Plate VIII, Fig. 2.

Magnetron Oscillator

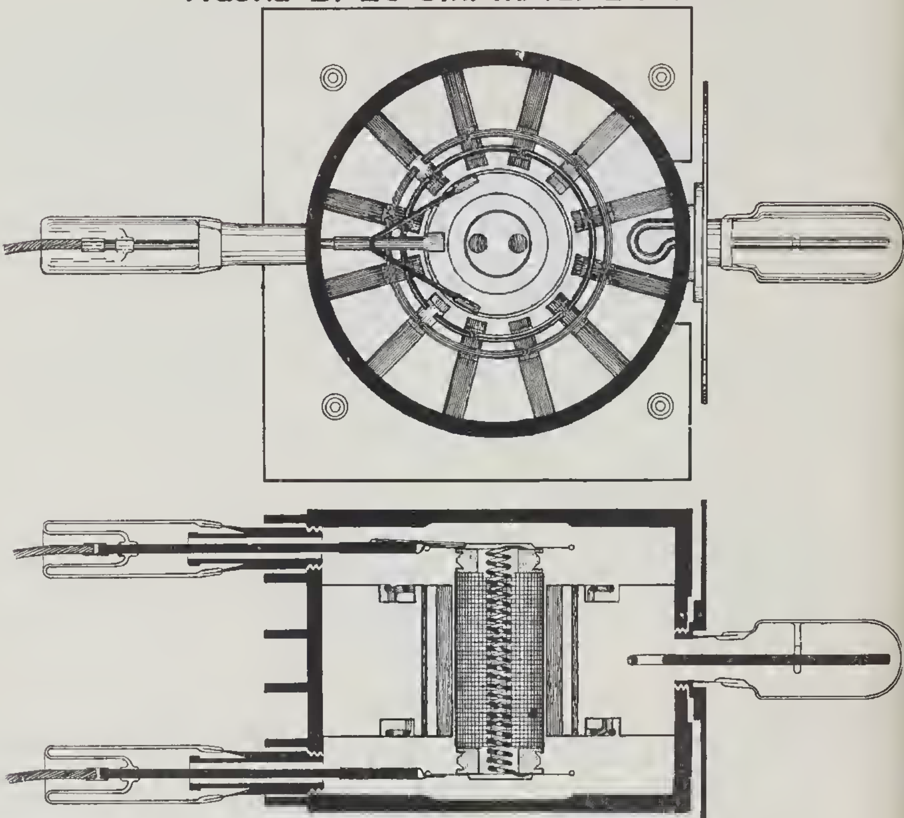
The 25 cm. magnetron was developed in 1944 at the CSIR Valve Laboratory, Melbourne University, for use in connection with L-band radar. As no account of this development has been published and as the magnetron forms the basis of the present accelerator a brief description of it will be given.

The magnetron is of the strapped vane-type containing 12 cavities. Its constructional features may be seen from Fig. 2. Operated at a field of 1100 gauss and a pulsed potential of -27 kV on the cathode, the magnetron is capable of delivering 500-600 kW peak power with an efficiency of 60%.

A set of normal operating characteristics is shown in Fig 3.

The design of the magnetron was at the time determined mainly by considerations of materials available. Thus the external copper cylinder for the

FIGURE 2. 25 CM. MAGNETRON..



body of the tube was the only appropriate material available in 1944. The selection of a vane-type was similarly determined by materials and workshop facilities. The assembly of the vanes to the body introduced a novel technique which might be mentioned briefly. The twelve vanes were assembled in a jig and turned in a lathe so as to fit the inner diameter of the body. Both the vanes and body were separately silver plated and then again tightly fitted together. Brazing flux was applied to the silver plating and the whole heated until the silver flowed so as to weld the vanes to the body.

The circuit diagram of the magnetron modulator used in the accelerator experiments is shown in Fig. 4. It consists primarily of a half-wave rectifier V_1 and filter circuit supplying a steady potential of 14kV. This potential is doubled by a resonant charging choke and hold-back diode V_2, and is then used to charge the two condensers C in parallel. These condensers are discharged in series by the rotary spark gaps G_1 and G_2 through a shaping-line and the magnetron load. The resulting square-topped pulse on the magnetron cathode is of 1 microsecond duration and of peak potential -27kV. The repetition frequency is 100 pulses per second. In order to synchronise the growth of the dynamic field with the start of the magnetron oscillation the field current to the coils is also operated by the magnetron modulator. Thus the modulator potential supply is used to charge the condenser K and the spark then shorts the condenser through the coils RR.

FIGURE 3. MAGNETRON CHARACTERISTICS

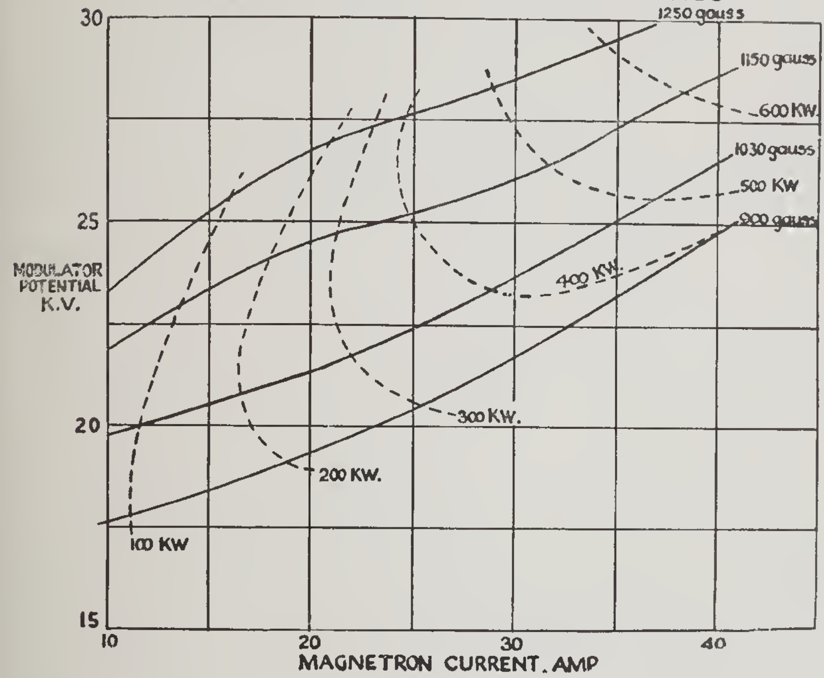
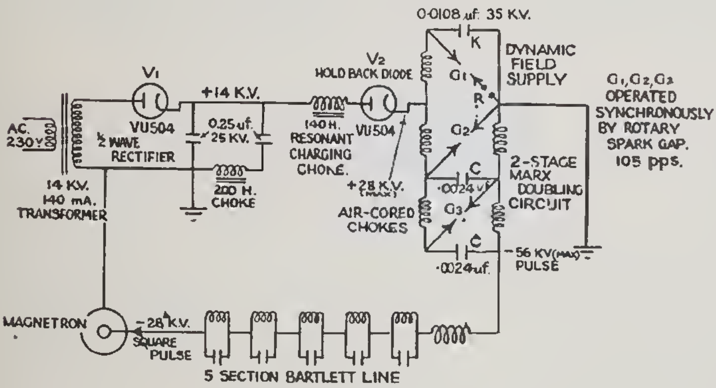


FIGURE 4. MODULATOR CIRCUIT



A problem in feeding the dees arises from the fact that the required radius of the dees is of the same order of magnitude as a quarter-wavelength of the oscillations. It appears therefore as though the dees might form a pair of centre-fed dipoles with the ends 90° out of phase with the centres. Such a phase variation would, of course, affect the acceleration of the electrons in their orbits. The whole problem is rendered theoretically difficult by the complicated structure of the dees themselves and probably experimental measurements of the voltage distributions along the dees would be the simplest approach to a satisfactory solution. In the absence of any precise information on this point, resort was had to spreading the Lecher wire feeding system into a parallel pair of fans. This probably will keep at least one edge of the dees in the same phase.

There was also the problem of measuring the dee voltages in the present accelerator. One of the best methods would have been to insert probes along the feeder line but these could not be easily provided for in the vacuum system. A qualitative estimate of the order 1000 V was made by noting the length of glow excited in a neon glow tube. An estimate of the order of magnitude can also be obtained theoretically in the following manner.

The magnetron in the accelerator experiments was operated at peak powers of 100 to 200 kW. In the waveguide, of section 7·5 cm. by 19 cm., the electric field vector E (volt/metre), given by the expression ;

$$E = \sqrt{\frac{4P \cdot 377}{ab \cos \theta}},$$

has the value 1·7 kV/cm. $\left\{\right.$ P (watt) is the total power transmitted as a $TE_{0,1}$ wave, $a = 7{\cdot}5$ cm, $b = 19$ cm, and $\cos\theta = \sqrt{(1 - (\frac{\lambda}{2b})^2)}$. $\left.\right\}$ In adjusting the dees to have a maximum voltage, a standing wave is set up in the guide so that a voltage antinode is located at the probe leading to the dees. Although it is not known to what extent the probe distorts the waveguide field it may be expected that a 5 cm. length probe would certainly pick up a greater potential than the standing wave voltage per cm. The probe voltage is transmitted along the feeder system to the dees which correspond approximately to a capacity termination. If the feeder system has a characteristic impedance of 100 ohm and the termination a capacity of 2 $\mu\mu$F, the calculated dee voltage is 1·15 times the probe voltage. Thus it can certainly be said that the dee voltage is greater than about 2000 to 3000 V.

Magnetic Field Coils

In the design of cyclotrons it has been found that the magnetic field should decrease slightly towards the outer edge so that magnetic focussing forces on the ions may be brought into being. These forces confine the ions to move in a stable central orbital plane and are of importance mainly at the outer orbits. If the difficulty of relativity mass increase of the ions is to be overcome by an increase of magnetic field for an increase of ion energies then it cannot be overcome by a static magnetic field which at the same time is focussing. In the synchrotron design, however, the magnetic field may be increased in step with the accelerated ions and the field may then always satisfy the form required for focussing.

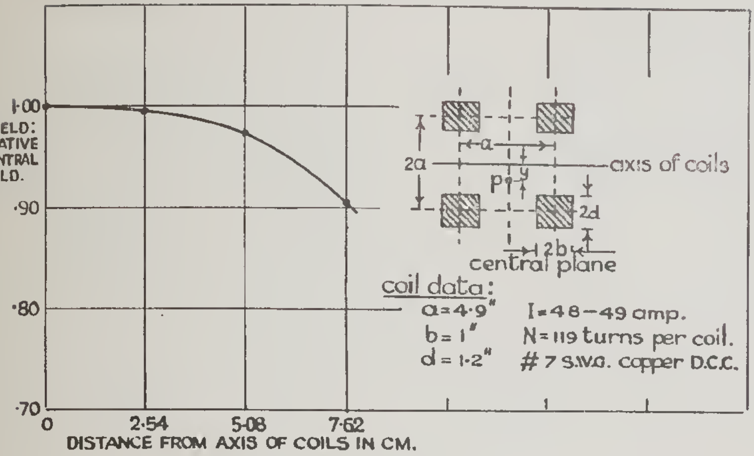
The form of the magnetic field was arbitrarily selected in the present accelerator to be that of a Helmholtz coil system. For this system of coils the form of field is readily calculable. The field F gauss at a point P in the median plane distant y from the axis is given by :

$$F = \frac{32\pi NI}{50\sqrt{5}.a}\left\{\left(1 - \frac{1}{15} \cdot \frac{d^2}{a^2}\right) + \frac{2^5}{5^3} \cdot \frac{y^2}{a^4}\,(36b^2 - 31d^2) - \frac{2 \cdot 3^3}{5^3} \cdot \frac{y^4}{a^4}\right\}$$

where N is the number of turns per coil and I is the energizing current in amperes.

For the coil data shown in Figure 5, the calculated field as a function of y is as shown. The field at the maximum orbit of the present generator is $1\frac{1}{2}\%$ less than the central field.

FIGURE 5. STATIC FIELD COIL DATA.



Another point of interest in connection with the static field coils arises since the dynamic magnetic field links with the static field coil. It is thereby essential to prevent the power in the dynamic circuit from being transformed into power in the static circuit. This was most easily accomplished in this case by direct shielding of the static coils. The whole static circuit was thoroughly shielded with sheet copper and further precautions of filtering were taken to prevent any high potentials entering the D.C. generator which supplied the static field current.

The dynamic coils are required to produce the maximum dynamic field within the time of the accelerating field pulse (1 μsec). One method of producing such a field is to short a charged condenser through a high-Q coil. The current i in the coil at an instant t after the initiation of the discharge is given by :

$$i = \frac{V_0}{L} \cdot \frac{e^{-bt}}{\sqrt{k^2 - b^2}} \cdot \sin \sqrt{k^2 - b^2} \cdot t$$

where V_0 is the initial potential of the condenser, L and b are the inductance and resistance of the coil and leads, and $k^2 = \frac{1}{Lc}$ where c is the capacity of the condenser. When $b \ll 1$, as in the present case ;

$$i \doteqdot \frac{V_0}{L} \cdot \frac{\sin kt}{k}$$

where the factor k is determined by the duration τ of the accelerating field pulse ; or ;

$$\frac{2\pi\sqrt{Lc}}{4} \doteqdot \tau \quad \text{i.e. } k = \frac{\pi}{2\tau}$$

The maximum current i is therefore proportional to V_0/L, and, for a particular voltage V_0, large currents are obtained for small inductance coils. Now the magnetic field is proportional to ni, where n is the number of turns per coil, and as L is proportional to n^2, the largest dynamic fields will be obtained for the coil of the smallest number of turns.

The starting point in the design was the magnitude of the largest condenser capacity that could be tolerated. The power available from the modulator imposed a limit on the capacity of the condenser to approximately 0·01 microfarad. Thus for $\tau = 1$ μsec, the magnitude of the inductance required was 10 microhenry.

Empirical calculations were made for a large number of coils of various geometrical and winding constants, also taking into account copper losses and proximity effects due to shielding. Owing to the relative certainty with which these calculations can be made for single layer coils, the final coils chosen were of single layer. Details of the present dynamic coils are: number of turns per coil $= 4$, diameter $5\frac{1}{2}''$, conductors, $\frac{1}{8}''$ copper tubing, separation between turns $= \frac{1}{8}''$, separation between centres of coils $= 2\frac{3}{4}''$.

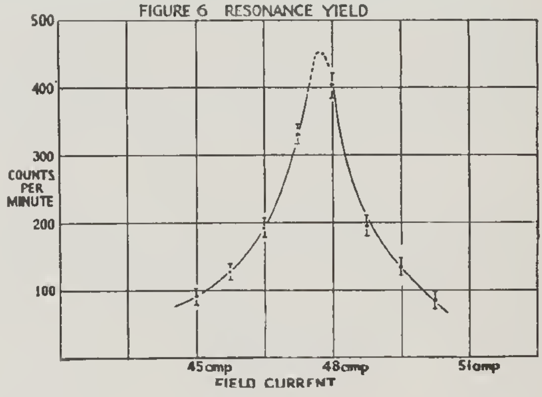
The field has been studied by means of a search coil and cathode ray oscillograph and for $V_0 = 20$ kV the measured peak induction was 270 gauss and the frequency of oscillation $= 430$ kilocycle/sec.

The rate of increase of the field B is approximately 1/40th that described by equation (10). The operation of the accelerator, therefore, corresponds to the synchrotron case and the energy added by the magnetic induction effect amounts to only 150 volts per turn.

Performance

The acceleration of electrons has been proved by using a geiger-counter to detect the X-rays produced by the electrons impinging on the target. That the electrons follow orbits consistent with magnetic resonance acceleration is proved by the existence of a resonance yield of X-rays at the value calculated from relation (2). The form of the resonance yield is shown in Figure 6.

FIGURE 6 RESONANCE YIELD



The quality of the X-rays has been investigated by absorption measurements in lead, iron and aluminium sheets. Typical absorption curves in lead are shown in Figures 7 and 8. The mean absorption co-efficients are consistent with a radiation energy of approximately 60 kV.

The yield of X-rays has been compared with the unfiltered radiation from a 1·4 milli-curie radium source and found to have a value 0·1 milli-curie. Since the radiation is being emitted only for a maximum period of 1 μsec. every 0·01 sec, the continuous yield of 0·1 milli-curie is equivalent to a peak yield during the acceleration time of 1 curie.

The nature of the radiation spectrum to be expected from an accelerator of the present kind has not been theoretically studied. Such an analysis will involve firstly the difficult space charge problems of the nature of the bunch of electrons in a single half-cycle acceleration, and secondly considerations of the